Schedule	Monday
08:50-10:20	**Holger Kammeyer** *Introduction to L^2-Betti numbers*
coffee break	
10:40-12:10	**Robert Young** *The geometry of nilpotent groups I*
lunch	
16:30-17:30	**Alain Valette** *Groups acting on trees and the first L^2-Betti number*
dinner	
	Tuesday
08:50-10:20	**Vincent Emery** *Volumes of hyperbolic lattices I*
coffee break	
10:40-12:10	**Steffen Kionke** *The growth of Betti numbers and approximation theorems I: Lück approximation and its generalizations*
lunch	
16:30-18:00	**Robert Young** *The geometry of nilpotent groups II*
dinner	

	Wednesday
08:50-10:20 **coffee break** **10:40-12:10**	**Holger Kammeyer** *L^2-torsion and torsion growth in homology* **Vincent Emery** *Volumes of hyperbolic lattices II*
lunch	
	free afternoon
dinner	

	Thursday
08:50-10:20 **coffee break** **10:40-12:10**	**Steffen Kionke** *The growth of Betti numbers and approximation theorems II: The growth of mod p Betti numbers* **Haluk Sengün** *Torsion homology of arithmetic Kleinian groups I*
lunch	
16:30-17:30 **17:45-18:45**	**Stefan Wenger** *Isoperimetric characterization of non-positive curvature* **Enrico Leuzinger** *Filling functions for arithmetic groups*
	conference dinner

	Friday
09:00-10:30	**Haluk Sengün** *Torsion homology of arithmetic Kleinian groups II*
coffee break	
10:50-11:50	**Roman Sauer** *The approximation theorem for L^2-Betti numbers - past and future*
lunch	

ABSTRACTS

Vincent EMERY (Bern) :

Volumes of hyperbolic lattices I, II

The goal of this mini-course will be to survey some results about the set of volumes ("the volume spectrum") of hyperbolic lattices, i.e., lattices in the Lie group PO(n,1). We will mostly focus on the case $n > 3$, but we will also explain how (and why) the cases $n = 2$ and $n = 3$ distinguish from the former. We will also try to highlight the distinction between arithmetic and non-arithmetic lattices. The "local rigidity" of lattices (which we will recall) will play an important role in the presentation of the subject.

Reference:

[1] E.V. Vinberg, V.V. Gorbatsevich and O.V. Shvartsman - Discrete subgroups of Lie groups, in: Lie groups and Lie algebras II, Encyclopaedia of Math. Sciences, vol. 21, Springer (2000).

Holger KAMMEYER (KIT Karlsruhe) :

Part I: Introduction to L^2-Betti numbers

The trace in a group von Neumann algebra is an operator algebraic tool that has proven to be relevant for topologically motivated problems. We will explain how it helps to overcome algebraic difficulties when trying to introduce an equivariant version of the well-known Betti numbers: the "L^2-Betti numbers". After giving the formal definition for CW complexes and groups, we move on to present some key properties and results in the theory. As applications, we will see how L^2-Betti numbers provide a possible route to answer seemingly unrelated questions: the conjectures of Hopf and Kaplansky.

Part II: L^2-torsion and torsion growth in homology

Reidemeister torsion, defined in the 30s, was historically the first homoemorphism invariant able to distinguish homotopy types of manifolds. Its L^2-version, termed "L^2-torsion", was not studied until the early 90s. But in some sense it seems to be the "more canonical" invariant. This is for instance reflected in its astonishingly parallel behavior to the Euler characteristic; one is tempted to call "L^2-torsion" the "odd dimensional Euler characteristic". After giving the formal definition and first properties, we move on to explain why L^2-torsion conjecturally equals the exponential growth rate of torsion in homology along a tower of finite coverings. This phenomenon has been studied most extensively in the context of hyperbolic 3-manifolds and of lattices in semisimple Lie groups.

References:

[1] H. Kammeyer - Introduction to L^2-invariants, lecture notes, to appear on http://www.math.kit.edu/ kammeyer/, (2017).
[2] W. Lück - Approximating L^2-invariants by their classical counterparts, EMS Surv. Math. Sci. 3 (2016), 269-344.
[3] W. Lück - L^2-Invariants: Theory and Applications to Geometry and K-Theory, Springer-Verlag, Berlin, 2002.

Steffen KIONKE (Düsseldorf) : **The growth of Betti numbers and approximation theorems**

Part I: Lück approximation and its generalizations

Given a space and a tower of finite sheeted covering spaces, it is of interest to understand how homological invariants behave along the tower; investigations of this kind are subsumed under the term "homology growth". In these two lectures we discuss the asymptotic behavior of Betti numbers in towers of finite sheeted coverings of a fixed CW-complex of finite type.

We first consider the case of rational Betti numbers. The main result, which will be discussed in detail, is the approximation theorem of Lück which identifies the limit of the sequence of normalized rational Betti numbers as the corresponding L^2-Betti number. The approximation theorem has been generalized in several directions and we briefly consider approximation along Farber sequences and sofic approximation. This leads us to two famous open problems: the Approximation Conjecture and the Determinant Conjecture.

Part II: The growth of mod p Betti numbers

In contrast to the case of rational Betti numbers, the growth of mod p Betti numbers is hardly understood. In general it is unknown if the corresponding sequence of normalized Betti numbers converges at all and we lack even a conjectural description of the limit. Following Bergeron, Linnell, Lück and Sauer we discuss convergence results for pro p towers and study the case of so-called p-adic analytic towers of coverings. The theory of Iwasawa algebras enables us to deduce precise results in this situation.
If time permits we take a look at the first Betti numbers of finitely generated, residually finite groups. Several invariants are conjectured to be related: the L^2-Betti number, the Betti number gradients and the rank gradient. We indicate some of the open problems.

References:

[1] N. Bergeron, P. Linnell, W. Lück, R. Sauer - On the growth of Betti numbers in p-adic analytic towers, Groups Geom. Dyn. 8 (2014), 311-329.
[2], [3] as above.

Enrico LEUZINGER (KIT Karlsruhe) :

Filling functions for arithmetic groups

The n-dimensional filling function of a highly connected simplicial complex or a Riemannian manifold measures the difficulty to fill $(n-1)$-cycles by n-chains. I will present joint work with Robert Young. We show that (arithmetic) lattices in higher rank semisimple Lie groups are undistorted up to the rank. More precisely, they satisfy the same Euclidean filling functions as the ambient groups (or the associated symmetric spaces). This broadly generalizes a theorem of Lubotzky-Mozes-Raghunathan on distance functions and confirms various conjectures of Thurston, Gromov and Bux-Wortman.

Roman SAUER (KIT Karlsruhe) :

The approximation theorem for L^2-Betti numbers – past and future

We review Lück's approximation theorem for L^2-Betti numbers which expresses L^2-Betti numbers as a limit of ordinary Betti numbers. Then we discuss invariant random subgroups and show why they are the right language for far-reaching recent generalizations of the original approximation theorem

Haluk SENGÜN (Sheffield) :

Torsion Homology of arithmetic Kleinian groups I, II

The goal of this mini-course is to present the conjectures of Bergeron and Venkatesh on the asymptotic growth of torsion in the homology of arithmetic groups in the special case of arithmetic Kleinian groups. We will start the mini-course by a general discussion quaternion algebras and arithmetic groups arising from orders in quaternion algebras. We shall then consider the cohomology of arithmetic Kleinian groups and together with the action of Hecke operators. In order to motivate the importance of torsion in the homology of arithmetic groups, we will discuss the partly-conjectural connection between cohomology of arithmetic Kleinian groups and Galois representations. Afterwards we will state the conjecture of Bergeron and Venkatesh.

References:

For the construction of arithmetic Kleinian groups:

[1] H. Sengun - Torsion in the homology of arithmetic Kleinian groups, cf.
https://sites.google.com/site/mhaluksengun/notes
[2] K. Petersen - Arithmetic groups and Lehmer's Conjecture, cf.

http://www.math.fsu.edu/ petersen/publications.html
[3] C. Maclachlan, A.Reid - The arithmetic of hyperbolic 3-manifolds, Graduate Texts in Mathematics, 219, Springer-Verlag, New York, 2003.

For connections with Galois representations and automorphic forms:
[4] H. Sengun - Arithmetic aspects of Bianchi groups, Computations with modular forms, 279?315, Contrib. Math. Comput. Sci., 6, Springer, Cham, 2014.
[5] H. Sengun - Some applications of number theory to 3-manifolds theory, cf.
https://sites.google.com/site/mhaluksengun/notes

For the torsion homology growth issue:
[6] H. Sengun - Torsion in the homology of arithmetic Kleinian groups, cf.
https://sites.google.com/site/mhaluksengun/notes
[7] J. Raimbault - Torsion homology of three-manifolds, cf.
https://www.math.univ-toulouse.fr/ jraimbau/
[8] N. Bergeron - Torsion homology growth in arithmetic groups, cf.
https://webusers.imj-prg.fr/ nicolas.bergeron/Travaux.html

Alain VALETTE (Neuchâtel) :

Groups acting on trees and the first L^2-Betti number

This is joint work with Talia Fernos (Greensboro). For a countable group G, denote by $\beta_1^{(2)}(G)$ its first L^2-Betti number. We study how the group property "$\beta_1^{(2)}(G) = 0$" behaves under group actions on trees. For a group G acting co-compactly on a tree with vertex-stabilizers G_v satisfying $\beta_1^{(2)}(G_v) = 0$, we completely characterize when $\beta_1^{(2)}(G) = 0$. This happens in particular when all edge-stabilizers are infinite.

Stefan WENGER (Fribourg) :

Isoperimetric characterization of non-positive curvature

The Dehn function, also known as the filling area or isoperimetric function, measures how much area is needed to fill closed curves of a given length in a space or a group by disc-type surfaces. In this talk we show that a locally compact geodesic metric space has non-positive curvature in the sense of Alexandrov (i.e. is a CAT(0)-space) if and only if its Dehn function is bounded from above by the Euclidean Dehn function on all scales. We furthermore provide a large scale analog which shows that if the Dehn function of a locally compact geodesic metric space has the same asymptotic growth as the Euclidean Dehn function then all its asymptotic cones are CAT(0). This is new even in the setting of Riemannian manifolds and in particular establishes the borderline case of a result about the sharp isoperimetric constant which implies Gromov hyperbolicity. Partially based on joint work with A. Lytchak.

Robert YOUNG (NYU Courant) :

The geometry of nilpotent groups

Gromov's famous *Polynomial Growth Theorem* showed that a group has polynomial growth if and only if it is virtually nilpotent. In this minicourse, we will study the geometry and analysis of such groups, with emphasis on the relationship between the group and its asymptotic cone. After introducing nilpotent groups and asymptotic cones and some of their basic properties, we will take a look at some topics of current research, such as Dehn functions and filling inequalities; surfaces and geometric measure theory; and embeddings and metric geometry.

References:

[1] C. Drutu - Quasi-isometry invariants and asymptotic cones, Internat. J. Algebra Comput. 12 (2002), no. 1-2, 99-135.

[2] B. Franchi, R. Serapioni, F. Serra Cassano - Rectifiability and perimeter in the Heisenberg group, Math. Ann. 321 (2001), no. 3, 479-531.
[3] M. Gromov - Carnot-Carathéodory spaces seen from within sub-Riemannian geometry, 79-323, Progr. Math., 144, Birkhäuser, Basel, 1996.
[4] M. Gromov - Groups of polynomial growth and expanding maps, Inst. Hautes Études Sci. Publ. Math. No. 53 (1981), 53-73.
[5] D. V. Osin - Subgroup distortions in nilpotent groups, Comm. Algebra 29 (2001), no. 12, 5439-5463.
[6] P. Pansu - Métriques de Carnot-Carathéodory et quasiisométries des espaces symétriques de rang un, Ann. of Math. (2) 129 (1989), no. 1, 1-60.
[7] S. Wenger - Nilpotent groups without exactly polynomial Dehn function, J. Topol. 4 (2011), no. 1, 141-160.
[8] S. Wenger, R. Young - Lipschitz homotopy groups of the Heisenberg groups, Geom. Funct. Anal. 24 (2014), no. 1, 387-402.
[9] R. Young - Filling inequalities for nilpotent groups through approximations, Groups Geom. Dyn. 7 (2013), no. 4, 977-1011.