From Lattice-Ordered Groups to Residuated Lattices: Hamiltonian and Nilpotent Varieties

Almudena Colacito

University of Bern, Switzerland

Nilpotent lattice-ordered groups (nilpotent ℓ-groups, for short) of class $c \in \mathbb{N}$ are those ℓ-groups with shortest central series of length at most c, and form a variety that can be defined relative to the variety of ℓ-groups by a semigroup equation (see, e.g., [4, 6]). Hamiltonian ℓ-groups are those for which every convex subalgebra is normal. The connection between these two classes goes beyond the fact that they can be seen as generalizations of the variety of Abelian ℓ-groups. It was shown in [3] that every nilpotent ℓ-group is Hamiltonian (cf. [8]), and a negative answer to the question whether the variety generated by all nilpotent ℓ-groups is the largest Hamiltonian variety was given in [1].

In this joint work with Davide Fazio and Constantine Tsinakis, we exploit well-known results about nilpotent and Hamiltonian ℓ-groups to argue for similar results about nilpotent and Hamiltonian prelinear cancellative residuated lattices. More precisely, we start by giving a positive answer to the question whether there exists a largest variety of Hamiltonian prelinear cancellative residuated lattices. Later, we extend work done in [5], and provide a categorical equivalence between nilpotent cancellative residuated lattices and nilpotent ℓ-groups endowed with a conucleus. By means of this result, nilpotent prelinear cancellative residuated lattices are proved to be semilinear and Hamiltonian. Finally, relying on results from [7, 2], we prove the failure of the amalgamation property for the variety of nilpotent class-c prelinear cancellative residuated lattices.

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