ORDERS ON GROUPS, AND SPECTRAL SPACES OF LATTICE-ORDERED GROUPS

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The spectral space of a lattice-ordered group is defined as the set of its prime convex sublattice subgroups with the hull-kernel topology. This notion of spectral space was initially introduced for Abelian lattice-ordered groups by Klaus Keimel in his doctoral dissertation, following on from the success of scheme theory in algebraic geometry. For a lattice-ordered group, its spectrum can also be thought of as the Stone dual of the distributive lattice (with 0) of its principal convex sublattice subgroups and, as such, it is a generalized spectral space. Here, the term generalized spectral refers to a space that is not necessarily compact but satisfies all other properties in Hochster's definition of spectral space. Moreover, spectra of lattice-ordered groups are completely normal, in the sense that the specialization order associated to the topology is a root system.

In 2004, Adam Sikora's "Topology on the spaces of orderings of groups" pioneered a different perspective on the study of the interplay between topology and ordered groups, that has led to applications to both orderable groups and algebraic topology. The basic construction in Sikora's paper is the definition of a topology on the set of right orders on a given right orderable group. The topology is then proved compact, Hausdorff, and zero-dimensional.

The theory of lattice-ordered groups and the theory of right orderable groups have been proved to be deeply related, and examples of this interdependence can be found almost everywhere in the literature of either field. E.g., every lattice-ordered group is right orderable as a group, and its lattice order can be obtained as the intersection of some of its right orders. For this reason, the question whether a relation can be found between the topological space of right orders of a right orderable group, and the spectrum of some lattice-ordered group arises naturally.

This talk will be based on joint work with Vincenzo Marra, where we provide a positive answer to this question. In order to give a satisfying result that intrinsically relates the two topological spaces, we employ a fully general construction involving all the varieties of lattice-ordered groups. So as to talk about the whole spectrum on the lattice-ordered group side, it is necessary to consider the broader notion of (right) pre-order on the group side. The resulting correspondence entails a few immediate consequences, including the possibility of representing certain lattice-ordered groups via the space of (right) orders on a group. If time allows, we will discuss some applications, and possibilities for further research.

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