Sequent Calculi for Logics of Agency: the Deliberative STIT

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Abstract

Dstit (deliberately seeing to it that) is an agentive modality usually semantically defined upon indeterminist frames – a semantics that builds upon a combination of Prior-Thomason-Kripke branching-time semantics and Kaplan's indexical semantics – enriched with agency. The temporal structure for branching time (BT) is given by trees with forward branching time, corresponding to indeterminacy of the future, but no backward branching, corresponding to uniqueness of the past. Moments are ordered by a partial order, reflecting the temporal relation, and maximal chains of moments are called histories. The trees are enriched by agent's choice (AC), a partition relative to an agent at a given moment of all histories passing through that moment (a partition since, intuitively, an agent's choice determines what history comes about only to an extent).

In such (BT+AC) frames, formulas are evaluated at moments in histories. Specifically, an agent a deliberately seeing to it that A holds at the moment m of a history h, holds iff (i) A holds in all histories choice-equivalent to h for the agent a, but (ii) doesn't hold in at least one history that m is a part of. In simple terms, the agent sees to it that A if their choice brings about those histories where A holds, but nonetheless it could have been otherwise (i.e. an agent can't bring about something that would have happened anyway).

While the semantics for stit modalities and logics built upon them is well-established, their proof theory has been largely restricted to axiomatic systems (starting with [18] and [2]) with just a few exceptions, mainly a treatment of multi-agent *deliberative* stit logic through labelled tableaux in [15] that builds upon Belnap's original semantics, and of the related *logic of imagination* in [17] that exploits a newly defined neighbourhood semantics, introduced in [16].

As for the meta-theoretical properties of stit logics, as for other logics, completeness is usually established through the method of canonical models for axiomatic systems and through exhaustive proof search for tableaux [17]. Decidability, on the other hand, has been achieved through filtration methods [18, 1].

Our aim in this work is to lay down the bases for the development of systems of deduction that cover the stit modalities presented by Belnap et al. [2], starting with dstit, in a way that respects all the desiderata of good proof systems, in particular to achieve a direct proof of decidability though a bound on proof search in a suitable analytic proof system.

Here the method of labelled sequent calculi developed since [6] is utilized: relatively complex truth conditions can be transformed into rules with the help of auxiliary modalities, as in the treatment of Lewis' counterfactuals [13], and additional properties for the characteristic frame conditions are expressed as sequent calculus rules following [9, 10]. The result is a G3-style labelled sequent calculus which is shown to possess all the desired structural properties, including being contraction- and cut-free.

Moreover, we demonstrate multiple applications of the system. We prove the impossibility of delegation of tasks among independent agents, the interdefinability of Dstit with an agent-relative modality *cstit* and an agent-independent modality *settled true*, as well as the treatment of refraining from [2] and [14]. Finally, we demonstrate the meta-theoretical properties of our system, namely soundness, completeness and decidability via a bounded proof search.

Keywords: BT+AC frames, labelled sequent calculus, deliberative STIT

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