Reverse Mathematics of some principles related to partial orders

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Reverse Mathematics is an ongoing program in mathematical logic, the main goal of which is to investigate the role of set existence axioms in the development of mathematics (a standard reference is [4]). Its standard setting is the theory of second-order arithmetic, Z_2. The goal of Reverse Mathematics is achieved by reversing the usual mathematical process, trying to deduce from a theorem the set existence axioms used to prove it.

In this talk, we will study (some variations of) the following theorem, due to Rival and Sands (see [3]) in the context of Reverse Mathematics:

Theorem. *(*RS-po*) Let P be an infinite partial order of finite width k. Then, there is an infinite chain C of P such that for every element $p \in P$, p is comparable with 0 or infinitely many elements of P.*

In particular, we will show that ACA_0, the third of the Big Five subsystems of Z_2, is enough to prove RS-po, although no reversal is known to hold. An interesting result is obtained by fixing the width of the partial order P: if $k = 3$, we proved that the theorem is equivalent to ADS, a combinatorial principle introduced by Hirschfeldt and Shore in [2], and a widely studied element of the "zoo below RT^2_2" (a very good presentation of which is given for instance in [1]). Notably, this version of the theorem appears to be the first natural mathematical statement proven to be equivalent to ADS.

This is a joint work with Marta Fiori Carones[1], Alberto Marcone[2] and Paul Shafer[3].

References

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