

FINITE MODELS AND PRACTICAL DECISION PROCEDURES IN PROPOSITIONAL SUBSTRUCTURAL LOGICS I, II, III

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Abstract: Residuated lattices are the algebraic models of propositional substructural logics, which are themselves non-classical logics determined by Gentzen-style sequent calculi that omit some of the structural rules. The first talk will review some of the basic definitions and examples within the algebraic setting, concentrating on the variety of residuated lattices and some of its most prominent subvarieties: generalized Basic Logic algebras, lattice-ordered groups, Heyting algebras, MV-algebras, Gödel algebras, product algebras and Boolean algebras. These varieties are related to their logical counterparts via standard techniques of algebraic logic to obtain generalized Basic Logic, ℓ -group logic, intuitionistic logic, Łukasiewicz logic, Gödel logic, product logic and classical propositional logic. The talk will review some of the standard decision procedures for these logics, and introduce a dual category of residuated frames that provides Kripke semantics for these varieties.

In the second talk we will cover some computational tools that are useful for implementing decision procedures and for searching for finite models. We will consider automated theorem provers and model finders such as Prover9/Mace4, SAT-solvers, constraint satisfiers, linear programming packages and SMT-solvers (SMT = satisfiability modulo theories). The latter tools are particularly well suited for interpreting the equational theories of abelian ℓ -groups, MV-algebras, Gödel algebras, product algebras and BL-algebras, thus giving practical decision procedures for many of the standard fuzzy logics. We will take a detailed look at implementing a specific translation from the equational language to SMT-LIB2, a standard language for SMT-solvers.

The third talk considers some algebraic techniques for deciding quasiequational theories, such as the finite embeddability property. The poset product construction will be used to describe all finite generalized BL-algebras, and a construction of kites gives useful examples of infinite non-linear models. We will also discuss some open problems, such as the decidability of generalized basic logic, and possible decision procedures for poset products of the unit interval MV-algebra over a finite poset.