Residuated frames for substructural logics

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Abstract. We start with an introduction to substructural logics, their algebraic semantics, i.e., (nonassociative) residuated lattices, and some basic proof theory. We then present joint work with Nick Galatos on residuated frames that provide Kripke semantics for residuated lattices as well as an algebraic view of Gentzen calculi and display calculi. We show how residuated frames are used to prove the finite model property and/or the finite embeddability property for many substructural logics or varieties of residuated lattices, including adaptations for involutive, distributive or semilattice logics. Correspondence theory and the Ackermann Lemma based algorithm ALBA are used to translate some propositional substructural formulas to first-order properties on the frames. We indicate how the methods developed for residuated frames generalize to (semi)lattices with operators, making them even more widely applicable. In addition to covering theoretical results, we also describe how these techniques can help find proofs or counterexamples in specific subvarieties using automated theorem provers and model finders.