YOUNG RESEARCHERS WORKSHOP ON SPECTRAL THEORY
BERN, OCTOBER 28–30, 2015

LIST OF SPEAKERS AND ABSTRACTS

1. **Sabine Bögli**
   Title: *Spectral approximation for unbounded linear operators with compact resolvents*
   Abstract: We study spectral convergence for sequences of (possibly unbounded) linear operators T_n, $n \in \mathbb{N}$, that converge to some operator T in strong resolvent sense. It is well known that, even in the case of purely discrete spectra, the eigenvalues of T_n may accumulate at a point that is not an eigenvalue of T. In addition to the occurrence of such spurious eigenvalues, for non-selfadjoint operators not every eigenvalue of T may be approximated. We prove that these two unwanted phenomena do not occur if the sequence of resolvents is so-called discretely compact. In addition, we present perturbation results for discrete compactness and for strong convergence of the resolvents. The theoretic results are applied to examples of non-selfadjoint Schrödinger operators in $L^2(\mathbb{R}^d)$ that are truncated to bounded but expanding domains in $\mathbb{R}^d$ (based on joint work with P. Siegl and C. Tretter).

2. **Jean-Claude Cuenin**
   Title: *L^p resolvent estimates for magnetic Schrödinger operators with unbounded background fields*
   Abstract: We prove L^p and smoothing estimates for the resolvent of magnetic Schrödinger operators. We allow electromagnetic potentials that are small perturbations of a smooth, but possibly unbounded background potential. As an application, we prove an estimate on the location of eigenvalues of magnetic Schrödinger operators with complex electromagnetic potentials.
   This is joint work with C.E. Kenig.

3. **Gian Michele Graf**
   Title: *The real spectrum of the imaginary cubic oscillator*
   Abstract: The imaginary cubic oscillator is the Schrödinger operator with potential $\mathrm{i}x^3$. Though being manifestly not self-adjoint, it is a paradigmatic example of $\mathcal{PT}$-symmetric quantum mechanics. Its spectrum is nevertheless real, as conjectured by Bessis and Zinn-Justin in 1992, later confirmed numerically, and finally proven by several people, e.g. by means of the Bethe ansatz. Here we will give an explanatory proof resting on WKB asymptotics and some common results in complex analysis (joint work with I. Giordanelli).

4. **Franz Hanauska**
   Title: *On the number of eigenvalues of linear operators on Banach spaces*
   Abstract: Let L_0 be a bounded operator and K a compact operator, both defined on a Banach space. We will estimate the number of discrete eigenvalues of $L := L_0 + K$ in subsets of the unbounded component of the spectrum of L_0, in terms of the approximation numbers of the perturbing operator K. Our method employs complex analysis and a finite-dimensional reduction, allowing us to avoid using the existing theory of determinants in Banach spaces, which would require strong restrictions on K.
   This is a a joint work with M. Demuth, M. Hansmann and G. Katriel.

5. **Marcel Hansmann**
   Title: *From spectral theory to bounds on zeros of holomorphic functions (and vice versa)*
   Abstract: In the first part of this talk (which is based on some joint work with G. Katriel), I will show how eigenvalue estimates for linear operators can be used to obtain new Blaschke type bounds on zeros of holomorphic functions on the unit disk. Conversely, in the second part of the talk, I will try to indicate how results about the distribution of zeros of holomorphic functions might be used to show that certain recently established eigenvalue bounds for linear operators (see the talk by Franz Hanauska) might be optimal.

6. **Raphael Henry**
Title: *Non-selfadjoint models in superconductivity*
Abstract: In this talk, we will recall the main spectral and pseudospectral properties of some simple non-selfadjoint models, and then present an application to the time-dependent Ginzburg-Landau model in superconductivity. More precisely, we will focus on the asymptotic behavior of some critical electric current in the case where the magnetic field is neglected.

7. **Vladimir Lotoreichik**
Title: *Spectra of definite type, tensor products, and non-self-adjoint waveguides*
Abstract: For $k = 1, 2$ let $\mathcal{H}_k$ be Hilbert spaces, $J_k \colon \mathcal{H}_k \to \mathcal{H}_k$ be bounded symmetric involutions, and $T_k \colon \mathcal{H}_k \to \mathcal{H}_k$ be m-sectorial J_k-self-adjoint operators. By standard argumentation the operator

$$T := \overline{T_1 \odot I + I \odot T_2} \tag{$\star$}$$

is m-sectorial in $\mathcal{H}_1 \otimes \mathcal{H}_2$ and J-self-adjoint with respect to $J = J_1 \otimes J_2$. We prove that some subsets of the spectrum of T are (resp., are not) of definite type, knowing a priori partial information on location of definite type spectra for T_1 and T_2.

Our motivation stems from the investigation of non-self-adjoint waveguides with various symmetries. In many cases Hamiltonians of these waveguides can be viewed as "small" perturbations of simple waveguide Hamiltonians having the structure as in ($\star$). Using our abstract machinery we show that some subsets in the continuous spectrum of the unperturbed waveguide Hamiltonian are of definite type. Further, employing well-established perturbation theory for spectra of definite type one can efficiently derive various conclusions on:

- reality of the spectrum,
- accumulation of non-real eigenvalues to real essential spectrum,
- pseudospectrum,

for perturbed waveguide Hamiltonians, which do not possess a simple tensor-product structure like in ($\star$), but which remain to be J-self-adjoint with respect to the same J. Most of these spectral conclusions are extremely hard to prove using direct methods, especially, when Green's function of the unperturbed Hamiltonian can not be explicitly expressed.

This talk is based on a joint work with P. Siegl.

8. **Boris Mityagin**
Title: *Systems $S(nx)$*
Abstract: After general discussion of the problem of completeness and basisness in $L^p(0, \pi)$ of this system for specific functions S,

$$S(x) = S(-x), \quad S(x + 2\pi) = S(x),$$

we focus on the case

$$S(x) = \sin x + \sum_{k=1}^{m} a_k \sin 2^k x, \quad a_m \neq 0.$$

The complete analysis is done in terms of the localization of zeroes of the polynomial

$$a(z) = 1 + \sum_{k=1}^{m} a_k z^k.$$

In a peculiar way, this analysis involves many known constructions (from A. de Moivre, 1712, to I. Schur, 1918, to harmonic analysis of the 1970's and 2010's).

9. **Radek Novák**
Title: *On the pseudospectrum of the harmonic oscillator with imaginary cubic potential*
Abstract: We study the Schrödinger operator with a potential given by the sum of the potentials for harmonic oscillator and imaginary cubic oscillator and we focus on its pseudospectral properties. A summary of known results about the operator and its spectrum is provided and the importance of examining its pseudospectrum as well is emphasized. This is achieved by employing scaling techniques and treating the operator using semiclassical methods. The existence of pseudoeigenvalues very far from the spectrum is proven, and as a consequence, the spectrum of the operator is unstable with respect to small perturbations and the operator cannot be similar to a self-adjoint operator via a bounded and boundedly invertible transformation. It is shown

that its eigenfunctions form a complete set in the Hilbert space of square-integrable functions; however, they do not form a Schauder basis.

10. **Frank Rosler**
Title: *A bound on the pseudospectrum of the harmonic oscillator with imaginary cubic potential*
Abstract: We are concerned with the non-normal operator

$$H = -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + ix^3 + cx^2 + bix$$

on $\mathrm{dom}(H) = \{\phi \in L^2(\mathbb{R}) \,|\, H\phi \in L^2(\mathbb{R})\}$, where $c > 0$, $b \geq 0$ are constants. It is well known that this operator is m-accretive and thus generates a one-parameter contraction semigroup e^{-tH}. Furthermore, it was shown by Dorey, Dunning, and Tateo in [1] (see also [2]) that the spectrum of H is real and positive. The ε-pseudospectrum of the operator, however, contains an unbounded set for any $\varepsilon > 0$ ([3, 4]) and thus does not approximate the spectrum in a global sense.

By exploiting the fact that the semigroup e^{-tH} is compact for $t > 0$, we show a complementary result, namely that for every $\delta > 0$, $R > 0$ there exists an $\varepsilon > 0$ such that

$$\sigma_\varepsilon(H) \subset \{z : \mathrm{Re}(z) \geq R\} \cup \bigcup_{\lambda \in \sigma(H)} \{z : |z - \lambda| < \delta\}.$$

In particular, the unbounded part of the pseudospectrum is contained in a half plane which moves towards $+\infty$ as ε decreases.

Furthermore, the semiclassical methods from [4] can be employed to show that for $c < 0$ no inclusion of the above type is possible. In fact, H does not even generate a bounded semigroup in this case.

Finally, we discuss aspects of the boundary case $c = b = 0$ which is still an open problem.

References

[1] P. Dorey, C. Dunning and R. Tateo. *Spectral equivalences, Bethe ansatz equations, and reality properties in PT-symmetric quantum mechanics*, J. Phys. A: Math. Gen, 34:5679-5704, 2001.
[2] K. C. Shin. *On the reality of the eigenvalues for a class of PT-symmetric oscillators* Commun. Math. Phys., 229:543-564, 2002.
[3] D. Krejčiřík, P. Siegl, M. Tater, and J. Viola. *Pseudospectra in non-Hermitian quantum mechanics,* ArXiv e-prints, February 2014 (arXiv:1402.1082).
[4] R. Novák. *On the pseudospectrum of the harmonic oscillator with imaginary cubic potential,* Int. J. Theor. Phys., 10.1007/s10773-015-2530-5

11. **Julien Royer**
Title: *Local energy decay and diffusive properties in a dissipative wave guide*
Abstract: We prove the local energy decay for the wave equation in a wave guide with dissipation at the boundary. It appears that for large times the dissipated wave behaves like a solution of a heat equation in the unbounded directions. The proof is based on resolvent estimates. Since the eigenvectors for the transverse operator do not form a Riesz basis, the spectral analysis does not trivially reduce to separate analyses in compact and Euclidean domains.

12. **Felix Schwenninger**
Title: *On results around the Inverse Generator Problem*
Abstract: *Is A^{-1} the generator of a bounded C_0-semigroup, if A is the generator of a bounded C_0-semigroup on a Banach space X and A^{-1} exists as a densely defined operator?* This question has become known as the *Inverse Generator Problem* and was studied first by de Laubenfels for analytic semigroups in the 80s. The answer is 'no' for general Banach spaces X, but remains open for general semigroups on Hilbert spaces X.
It is well known that the Inverse Generator Problem is closely related to the *Cayley Transform Problem*, i.e., the question whether the operator $(I+A)(I-A)^{-1}$ is power-bounded, if A generates a bounded C_0-semigroup (where I denotes the identity on X). In this talk, we give an overview on both problems and show that they are equivalent. Furthermore, it is shown that they can be reduced to the special case of exponentially stable semigroups.

13. **Albrecht Seelmann**
Title: *On an estimate in the subspace perturbation problem*
Abstract: We study the problem of variation of spectral subspaces for linear self-adjoint operators under an additive perturbation. The aim is to find the best possible upper bound on the norm

of the difference of two spectral projections associated with isolated parts of the spectrum of the perturbed and unperturbed operators in terms of the strength of the perturbation.

In our approach, we formulate a constrained optimization problem on a finite set of parameters, whose solution gives an estimate on the norm of the difference of the corresponding spectral projections. In particular, this estimate is stronger than the one recently obtained by Albeverio and Motovilov in [Complex Anal. Oper. Theory 7 (2013), 1389–1416].

14. **František Štampach**
Title: *Spectral analysis of a complex Jacobi matrix associated with Jacobian elliptic functions*
Abstract: We present some recent result concerning spectral properties of certain one-parameter family of Jacobi operators with vanishing diagonal. Although the operator from the family is non-symmetric in general, a lot of its spectral properties (spectrum, eigenvectors, Weyl m-function, etc.) can be found either fully explicitly, or in terms of special functions, namely, Jacobian elliptic functions. In addition, depending on the parameter, the spectrum of the operator possesses an interesting transition property.

15. **Joseph Viola**
Title: *Evolution equations with non-self-adjoint supersymmetric quadratic operators*
Abstract: In a joint work with A. Aleman, we consider exponentials e^{-tQ} for operators Q which are quadratic in (x, ∂_x) with $x \in \mathbb{R}^n$ which admit a supersymmetric decomposition; for simplicity we focus on the twisted harmonic oscillators of Davies, $Q = a(-(d/dx)^2 + bx^2)$ with $\operatorname{Re} b \neq 0$. Despite difficulties coming from the pseudospectrum and lack of a spectral resolution, we characterize boundedness and return to equilibrium of e^{-tQ} for all complex times t. These characterizations follow from a classical reduction to a transport operator using a Bargmann transform.